Simulation of Thickened Tailings Stacks

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ABSTRACT

An a-priori method is presented for predicting the slope of a non-segregating tailings slurry in an open channel. This method has been validated with experimental data collected at two mine sites. A new empirical model is also presented. Various channel shapes have been considered in order to enable the model to be applied for tailings beach slope prediction, and it was found that the shape had very little impact on the predictive ability of the model, thus resulting in the introduction of a robust a-priori tailings beach slope prediction method. A three dimensional tailings stack model is also presented, which offers realistic shape predictions and an explanation for beach concavity in non-segregating tailings.

1 INTRODUCTION

A non-segregating thickened tailings slurry forms its own turbulently flowing channel that runs down a tailings beach at an “equilibrium slope”, which is sufficient to keep all the slurry particles in motion, but not so steep as to erode the underlying beach (Winterwerp et al., 1990, Pirouz et al., 2005, Fitton et al., 2006a). This channel equilibrium slope dictates the slope of the tailings beach formed, which is itself dictated by the discharge rate and the slurry properties (Fitton et al., 2006c).

2 BEACH SLOPE PREDICTION

An a-priori equilibrium slope prediction method is presented, based on non-Newtonian open channel flow, sediment transport and rheology theory. To make a slope prediction with this method the following input data must be defined:

- Q, the flow rate (m^3/s).
- C_w, the concentration of the tailings slurry in terms of weight.
- d_{50}, the median particle diameter of the tailings slurry (m).
- d_{90}, the 90th percentile particle diameter of the tailings slurry (m).
- ρ_w, the density of the carrier fluid of the tailings slurry (m).
- ρ_s, the density of the solid particles in the tailings slurry.
- τ_y, K and n (Rheological parameters for a Herschel-Bulkley model fit for the applicable concentration of the slurry).

The channel cross-sectional shape needs to be defined in such a way that the cross sectional area of flow, A, and the wetted perimeter of the channel, P can be calculated as a function of the depth.

The slope of an open channel with uniform flow can be calculated with the Darcy-Weisbach equation:

$$S_0 = f \cdot \frac{V^2}{8gR_H} \tag{1}$$

where S_0 is the slope of the channel bed under uniform flow conditions, R_H is the hydraulic radius, V is the average velocity of flow, g is the acceleration due to gravity and f is defined as the Darcy friction factor.

The open channel form of the Colebrook-White equation will be used to calculate the Darcy friction factor:

$$\frac{1}{\sqrt{f}} = -2\log_{10}\left[\frac{k_s}{14.8R_H} + \frac{2.51}{\mathrm{Re}\sqrt{f}}\right] \tag{2}$$

Of key interest in this work is the k_s parameter, which represents the roughness of the pipe or channel walls (expressed in meters). The value of 2 x d_{90} has been adopted for estimating k_s on the basis of work done by Ikeda et al. (1988) and Fitton et al. (2006c).

The Re parameter in the Colebrook-White equation is the Reynolds Number. Haldenwang et al. (2004) presented the following equation for calculating the Reynolds number for Non-Newtonian turbulent open channel flow, with the rheological properties of a fluid of density ρ expressed in terms of the three Herschel-Bulkley fitting parameters τ_y, K and n:

$$\mathrm{Re}_{HB} = \frac{8\rho V^2}{\tau_y + K\left[\frac{2V}{R_H}\right]^n} \tag{3}$$

From numerous experimental studies into the transport of sediments in turbulent flows, there are many equations presented in the literature that enable the prediction of the minimum average velocity required to keep particles in suspension in a pipe (V_C). This particular velocity appears in the literature under several names, with the most common ones being “minimum transport velocity”, “deposition velocity” and the “critical velocity”. Fitton et al. (2006c) investigated the performance of 16 minimum transport velocity equations from the literature. It was found that the Durand equation with the Wilson and Judge correlation performed quite well, which is expressed below in an open channel form:

$$V_C = 2.0 + 0.3\log_{10}((d/4R_H)C_D)\sqrt{8gR_H(s-1)} \tag{4}$$

where d is the median particle diameter, C_D is the particle drag coefficient, and s is the ratio of the density of the solid particles to the density of the carrier fluid. A value of 0.45 for the particle drag coefficient (C_D) was assumed, which has been deemed a reasonable estimate by others; 0.45 in Ishii and Zuber (1979); 0.44 in Walton (1995).

2.1 A New Empirical Minimum Transport Velocity Model

An empirical minimum transport velocity model was developed by fitting two curves to the experimental data that was gathered at Cobar and Sunrise Dam; one for non-segregating slurries and the other for segregating ones. The equations for the two curves are presented below. The symbol K_{BP} refers to the Bingham plastic viscosity of the fluid.

$$V_{CNon-segregating} = 0.145\ln\left(\frac{\rho V R_H}{K_{BP}}\right) \tag{5}$$

$$V_{CSegregating} = 9.5\times10^{6} K_{BP} d_{50} \tag{6}$$

2.2 Equilibrium Slope Prediction Methodology

The steps involved in making a slope prediction are shown below. This method is ideally handled within a spreadsheet program, where the necessary iterative calculations are managed efficiently.

- Guess an initial value of the depth.

- Calculate the cross sectional area of flow, A, and the wetted perimeter, P, as a function of the geometry of the flume channel.
- Calculate R_H, the ratio of A/P.
- Calculate V, the average velocity in the channel (equal to Q/A).
- Calculate V_C, the minimum transport velocity.
- Calculate Re_{HB} using equation 3.
- Calculate f using equation 2, with Re_{HB} providing a value for Re.
- Calculate S_0 using equation 1.
- Adjust the initially guessed depth value until V and V_C in steps 4 and 5 equate. At this point, the slope calculated in step 8 will be predicted equilibrium slope.

3 EXPERIMENTAL DATA

The predictions have been compared with experimental data that was collected by the authors in two field campaigns conducted at the Peak gold mine in Cobar, New South Wales in June 2004 and at the Sunrise Dam gold mine near Laverton in Western Australia in February 2005. This experimental work has been described in some detail in Fitton et al. (2006a). A summarised description of the experimental work and the resultant data is still warranted here however.

The experimental work was centred on a 10 m long flume with a half pipe of circular cross section providing the channel shape. The internal surface of the half pipe was coated with sand. During the Cobar programme two different sized half pipes were used; radial dimensions were 207 mm and 180 mm respectively. During the Sunrise Dam campaign only one pipe was used, which had a radius of 170 mm. In both experimental campaigns the slope of the flume could be adjusted from 0% (horizontal) to 7%. Tailings slurry was run down the flume at various flow rates and concentrations, with a trial and error approach used to determine the prevailing equilibrium slope for a given discharge scenario. Rheological analysis was done with a viscometer. A summary of the data gathered during the two campaigns is presented in Table 1.

Table 1 Summary of experimental data recorded over the two programmes

Experimental measurement	Cobar	Sunrise Dam
No. of equilibrium slopes recorded	9	41
Steepest equilibrium slope (%)	3.7	6.0
Flattest equilibrium slope (%)	1.7	0.75
Maximum flow rate (l/s)	19	24
Minimum flow rate (l/s)	1.9	1.9
Maximum concentration (% w/w)	57.7	66.8
Minimum concentration (% w/w)	45.8	25.8
Segregation threshold (% w/w)	~ 46	~ 52
Herschel-Bulkley rheological fit for slurry at maximum concentration:		
Yield strength, τ_y (Pa)	3.65	21.2
Consistency index, K (Pa.s)	0.076	0.44
Power, n	0.81	0.60
Bingham plastic viscosity, K_{BP} (Pa.s)	0.016	0.030

4 RESULTS

The fit plots of the Durand-Wilson-Judge model and the new empirical model are presented in Figures 1 and 2 respectively, in terms of the predicted equilibrium slope vs the experimentally observed equilibrium slope. In the key of each plot the letters "SD" refer to Sunrise Dam.

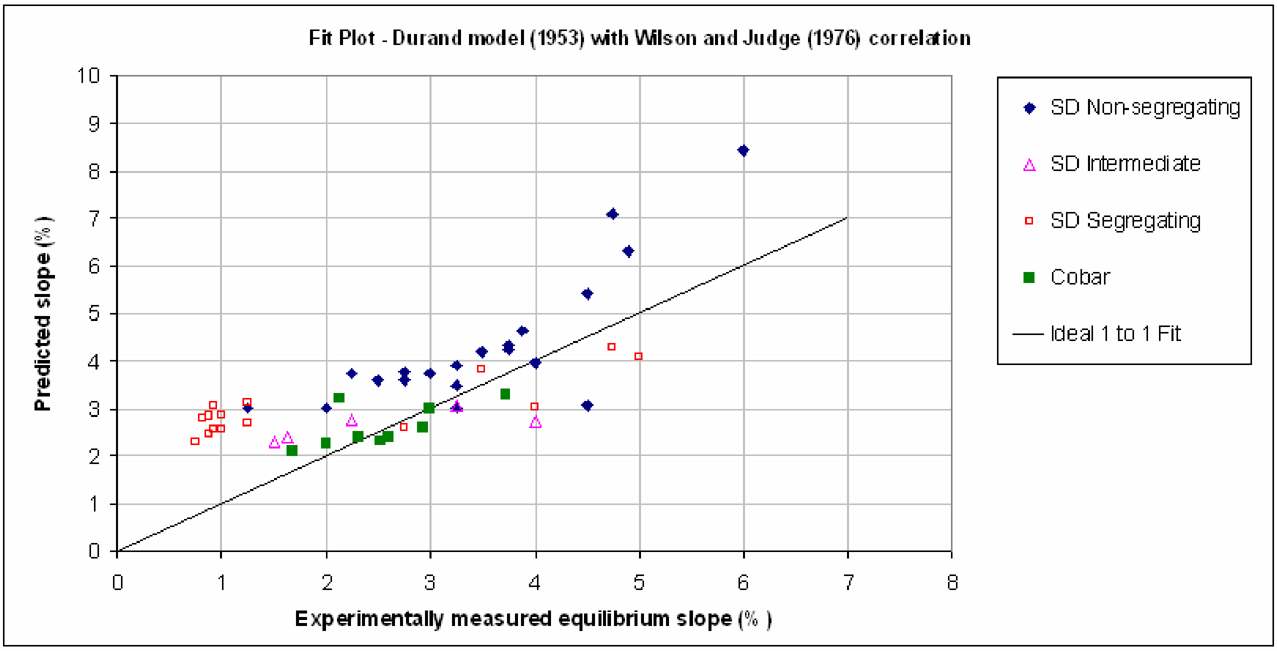

Figure 1 **Fit plot of Durand, Wilson and Judge model against experimental data**

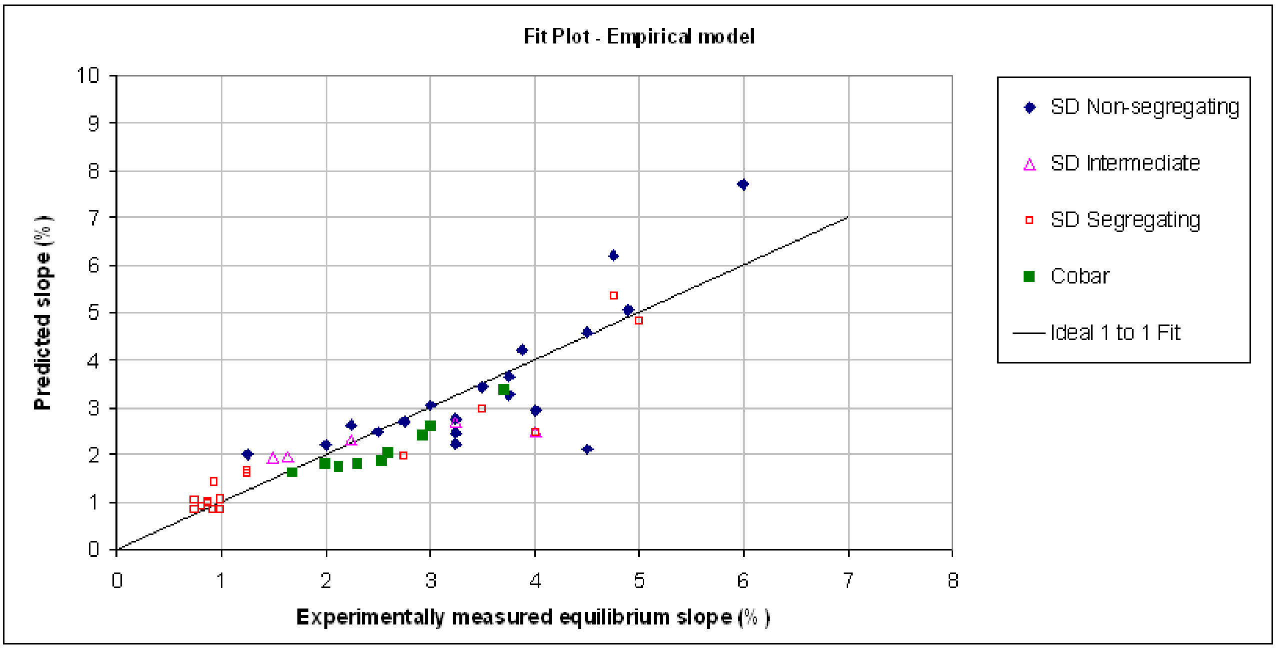

Figure 2 **Fit plot of the new empirical model against experimental data**

4.1 Channel Shape and Prediction of Beach Slope

In applying this slope prediction method to validate the experimental data, the channel cross-sectional shape was defined by the semi-circular channels used in the flume. However, to enable this method to predict tailings beach slopes, it is necessary to consider the cross-sectional shape of the self formed channels that occur on tailings beaches. This raises the following questions:

- What channel shape does the tailings actually adopt in nature?
- How can the model be modified to allow for this?
- How does this affect the predictive ability of the model?
- How sensitive is the model to different channel shapes?

The cross-sectional shape of a channel that forms when hydraulically discharged tailings erode the underlying deposited material can be related to other similar processes that occur naturally, such as river morphology, alluvial erosion processes, mud flows, lava flows and even in ice flows. A considerable amount of research has been made into the nature of these flows, with significant attention going towards the prediction of the channel shape formed. Parker (1979) presented an equation for predicting the cross-sectional shape of a particular type of self-formed alluvial river with a gravel bed, termed a "threshold canal", which took the form of the trough of a sinusoidal wave. His defining criterion for a threshold canal was one with bed material that was essentially at the threshold of being transported by the river water. His equation was based on the vector addition of forces acting on a gravel particle, but it was then demonstrated that there were some fundamental errors in the defining tenets of this approach, resulting in a paradox where it is possible for a so-called threshold channel to be stable with fluctuations in flow velocity, and likewise, for a so-called stable channel to be susceptible to erosion. Other workers have tried other approaches to avoid this paradox as discussed by the ASCE Task Committee on Hydraulics (1998), but it is concluded that the prediction of river bed shapes is still not fully understood.

An article was presented recently by Chryss et al. (2006) in which the shape of a tailings channel was predicted, also based on equating the forces at work at the channel boundary. The shape presented had a curved base, but due to the mathematical collapse of the model in the region of the channel banks, this forced the adoption of straight vertical surfaces to enable its application. Furthermore, for this model to be applied, the concentrations of the slurry and the underlying bed material are required as input parameters. Since the bed concentration is not known, it is not possible to apply the Chryss et al. model in this work without making some further study into the bed density of settled tailings. To complicate this, it is noted that the bed concentration will be constantly increasing as a result of the slow settlement of the underlying tailings during the time that follows the channel formation. In theory, this will cause the shape of the channel to gradually change over time to reflect this change in bed density. Further research is needed in this area if channel shape in a tailings beach is to be better understood.

In this work an analytical approach will be taken to test the impact of various artificial channel shapes. Several simple geometric shapes will be tested with width to depth ratios based on the actual shapes measured on naturally formed channels in tailings. This will enable a quantitative comparison between the various shapes to ascertain what effect each one has on the predicted slope. Figure 3 shows three naturally formed channel shapes superimposed over four artificial geometric shapes; those being 2 parabolas, a rectangle and a circle. The circular section is based on the actual dimensions of the flume used during the Cobar experiments.

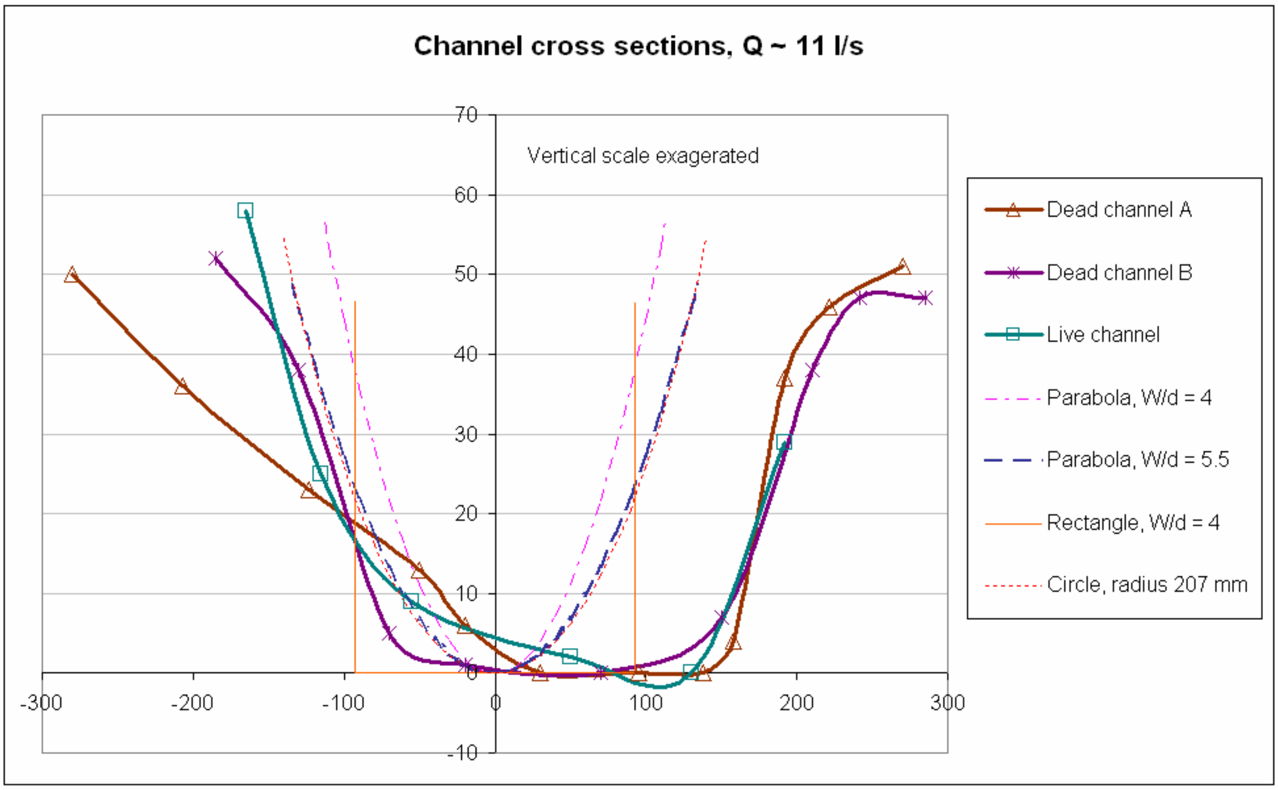

Figure 3 **Four artificial channel shapes superimposed over three self-formed channel shapes. In all cases, Q ~ 11 l/s**

For each of these four artificial shapes, slope predictions were made with each of the two minimum transport equations being used; the Durand-Wilson-Judge model and the new empirical model. In each case, the geometry of the shape was used to calculate the cross sectional area and wetted perimeter as a function of the depth in the same way as was done for the circular cross section. Equilibrium slope predictions were made using the same flow rates and slurry characteristics that were recorded in the 50 flow regimes from the Cobar and Sunrise Dam data sets, so that the predicted slopes could be compared to those measured experimentally in the flume. For each case the slope prediction method was done in exactly the same sequence as has been presented in section 2.2. The results of these predictions are presented in Table 2, with the sum of the absolute deviations between the measured and predicted points being adopted as an indicator of the quality of the fit (ie. the smaller the sum of the absolute deviations, the better the fit).

Table 2 **Statistical fit of four channel shapes run through the two slope prediction models**

Model	**Channel Shape**	**Sunrise Dam**			**Cobar**		**N.S. Tot.**	**Tot-al**
		N.S.	**Int.**	**Seg**	**N.S.**	**Seg**		
Durand-Wilson-Judge	Circular channel in flume	20.2	3.5	22.0	2.0	1.1	22.2	48.8
Durand-Wilson-Judge	Parabola, W/d = 5.5	20.3	3.7	22.1	2.3	0.8	22.6	49.2
Durand-Wilson-Judge	Parabola, W/d = 4	16.6	3.6	21.3	2.7	0.7	19.3	44.8
Durand-Wilson-Judge	Rectangle, W/d = 4	18.9	3.7	21.7	2.5	0.8	21.3	47.5
Fitton et al	Circular channel in flume	11.6	2.9	6.0	3.3	0.4	14.9	24.2
Fitton et al	Parabola, W/d = 5.5	10.1	3.2	7.1	3.2	0.7	13.3	24.3
Fitton et al	Parabola, W/d = 4	10.3	3.1	7.9	4.4	0.8	14.8	26.6
Fitton et al	Rectangle, W/d = 4	9.9	3.2	7.4	3.6	0.7	13.4	24.7

This exercise has led to some interesting findings:

- The cross-sectional shape had a minor effect on the prediction, which suggests that the model is robust with regards to channel shape.
- The width to depth ratio also had a low impact on the predicted channel slope than the cross-sectional shape itself. This would suggest that the model is also robust when width-to-depth ratios are concerned.

Both these findings raise interesting points about channel shape in general. In particular, the hydraulic radius of a channel is shown to be the only relevant characteristic of a channel in expressing of shape, which supports the findings of Haldenwang et al. (2004).

From this exercise, an important step has been taken to enable the model to be applied to the prediction of tailings beach slopes rather than just equilibrium slopes in channels, with the experimental data providing strong support for the validity of this approach.

From the statical results presented in Table 2 as well as the comparative channel shapes presented in Figure 3, it is recommended that the best shape approximation to be applied is the parabola with a width 5.5 times the depth.

5 STACK SHAPE PREDICTION

A tailings stack can be geometrically described as a mound of many thinly spread deposits of tailings, each caused by the supply of tailings to some location on the stack's surface by a self-formed channel. At the end of a channel the tailings can be seen to disperse in a wide flat sheet about 5 to 10 cm thick, but over time the channel gradually advances in length along the top of this freshly deposited sheet, enabling the tailings to gradually spread further from the stack's apex. The direction taken by a self-formed channel is essentially dictated by gravity as the preceding sheet flow tailings runs along the path of least resistance. Eventually the channel experiences a bank up effect when the sheet flow builds up in an area that is slightly flatter than the channel slope, at which time the channel will suddenly adopt a completely new path from some point upstream. The prevailing channel slope at any instant is dependant on the flow rate and rheology of the slurry. As a result, a viscous slurry will deposit near the top of the stack at a steep slope, while a watery one will run further away at a flatter slope. Likewise, a low flow rate will see the channel run at a steep slope, with the deposition occurring near the apex of the stack, while a large flow rate will cause the channel to adopt a flatter slope, thus resulting in deposition further out.

In order to develop a three dimensional geometric model to simulate the process described above, each channel path has been assumed to run as a straight line that heads radially outward from the top of the stack in a random direction. A random number generator has been used to define the direction of the channel for each regime in the form of a compass bearing. It is acknowledged that straight channels do not form on a real tailings stack, but the random direction of each channel will ensure that the location of each deposit occurs in a realistically random location on the stack's surface.

The author developed a computer program that would conduct thousands of trial and error calculations to determine where each new deposit will lie. Each deposit was shaped as a simple right angled cone with a slope equal to the prevailing channel slope, and the computer determined an appropriate position for the apex of this cone that falls either along the path of the prevailing channel or otherwise above the apex of the stack, such that the volume trapped between the surfaces of the new cone and the underlying stack is equal to the discharged volume of that deposit.

The discharged volume of each deposit was calculated by multiplying the flow rate with the duration of the discharge, and the beach slope determined with the slope prediction method. The final location of each deposit fell into one of two categories. In the majority of cases it formed a section of a cone somewhere on the side of the stack, but in the case where the prevailing channel slope was relatively steep, the new deposit crowned the top of the stack. The computer model used logic statements to determine which of these two outcomes was appropriate, with each deposit shaped and located accordingly. In work presented in Fitton et al. (2006b), the accuracy of this three dimensional model was tested against actual survey data, using real discharge history for input data. A three-dimensional plot from this model is presented in Figure 4.

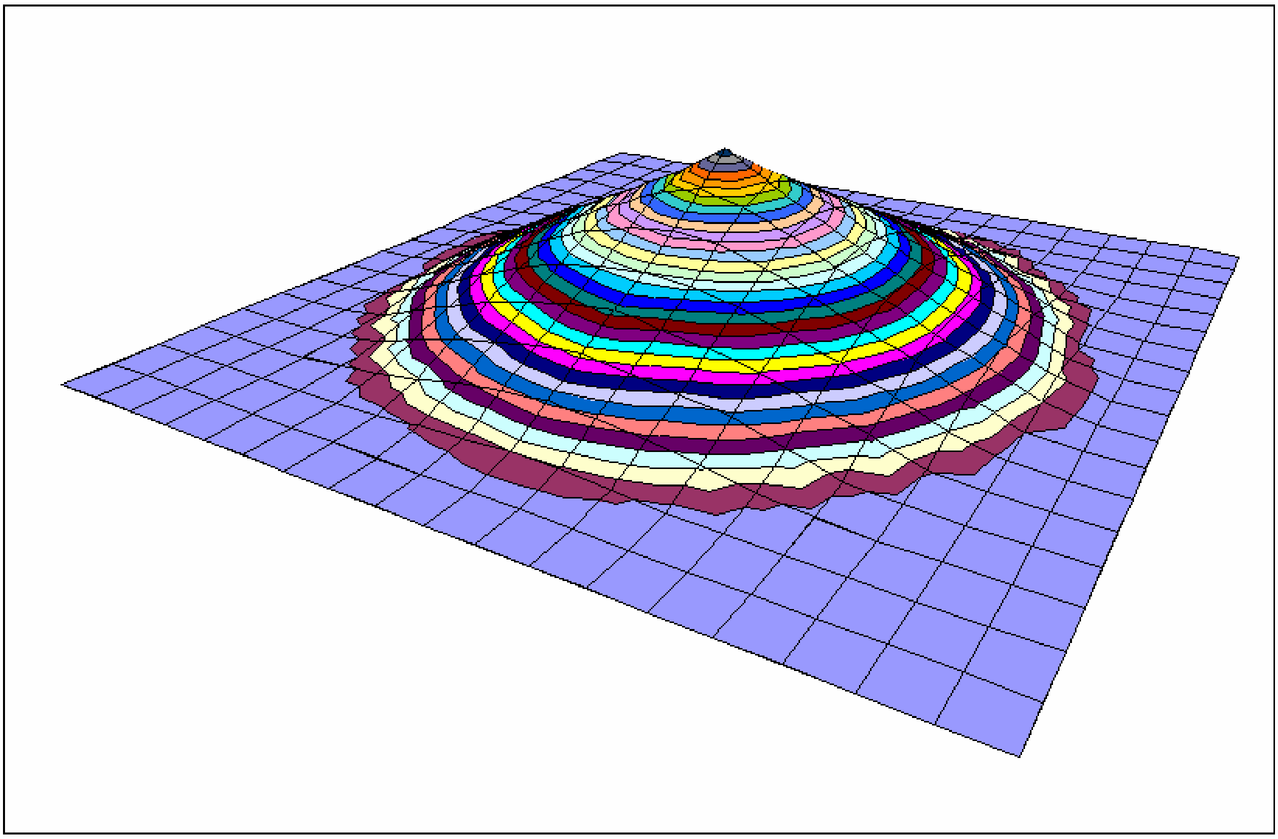

Figure 4 **A three dimensional plot of a simulated stack generated by the stack model**

It is noted that this model predicted concaved beach slopes on a large scale due to the compounding of different linear beach slopes of all the smaller deposits that formed the stack surface, with the steeper deposits being located at the top of the stack and the flatter ones at the toe. It could thus be shown that the concavity was caused by variations in the flow rate or rheology of the slurry. If uniform non-segregating tailings slurry were to be discharged at a constant flow rate for a long time, the stack would essentially resemble a perfect cone, but mineral processes constantly experience fluctuations in ore type, water content, equipment reliability etc, leading to changes in flow rate or rheology (or both), which leads to concavity. The greater the fluctuations in the discharged tailings, the greater the degree of concavity. This should not be confused with the segregating nature of more dilute slurries however, where it has been reported that particle sorting causes beach concavity (Blight, 1994; Morris, 2004).

6 CONCLUSIONS

An a-priori model has been presented here for predicting the equilibrium slope of a channel formed by the hydraulic discharge of a non-segregating tailings slurry. The experimental results provide significant validation of the model. A new empirical minimum transport velocity model has been presented. A robust channel shape approximation was presented to enable the prediction of tailings beach slopes, with validation lent to this approach by the experimental data. A three-dimensional shape prediction model for tailings stacks was then described, which applied the slope prediction model in simulating the growth of a tailings stack, and provided some realistic results with a well founded explanation for the concavity of a tailings beach formed by the discharge of a non-segregating slurry.

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